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Possible Connection among Parity, Strangeness and Isotopic Spin.

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Elementary particle models based on invariance under Lie groups have been proposed (1-4). The point of view we shall take here will be to consider such models as complete dynamical models for elementary particles.

Particularly, the interactions are formed out of the currents that are generated in the model by the group invariance.

Breakdown of the full invariance is assumed only to appear in the physically significant solution of the invariant dynamical equations.

Such general viewpoint is also expressed in works by Heisenberg (5), Nambu (6), Goldstone (7), and most recently by Baker and Glashow (8).

In such a framework we examine the possibility of a connection between parity conservation and conservation of strangeness and isotopic spin. We con-consider in detail the Sakata model, the octet model (also called eightfold-way) and the model based on G_2. We find that parity conservation implies conservation of strangeness and of isotopic spin. Conversely: strangeness conservation implies parity conservation and isotopic spin conservation in the Sakata model and in the model based on G_2; strangeness and isotopic spin conservation imply parity conservation in the octet model (it is assumed that the fundamental fields belong to irreducible representations of the Lie group in both their chiral components).

We first discuss in detail the Sakata model. The discussion will also illustrate the sense of our statements.

Under infinitesimal transformations of SU_3 the spinor ψ describing p, n, and Λ transforms as

$$\psi \to \left(1 + i\sum_{i=1}^{8} \varepsilon_i \frac{\Lambda_i}{2}\right)\psi\,, \tag{1}$$

(1) M. Ikeda, S. Oyawa and Y. Ohnuki: *Prog. Theor. Phys.*, **22**, 715 (1959).

(2) M. Gell-Mann: *Phys. Rev.*, **125**, 1067 (1962).

(3) Y. Neeman: *Nucl. Phys.*, **26**, 222 (1961).

(4) R. E. Behrends, J. Dreitlein, C. Fronsdal and B. W. Lee: *Rev. Mod. Phys*, **34**, 1 (1962).

(5) W. Heisenberg: *Annual Intern. Conf. on High-Energy Physics at CERN* (Genève, 1958).

(6) Y. Nambu: *International Conference on High-Energy Physics at Rochester* (Rochester, 1960), p. 858.

(7) J. Goldstone: *Nuovo Cimento*, **19**, 154 (1961).

(8) M. Baker and S. L. Glashow: *Phis. Rev.*, **128**, 2462 (1962).

where ε_i are infinitesimal and the 3×3 matrices $\frac{1}{2}\Lambda_i$ constitute a representation of the infinitesimal generators F_i of SU_3.

The generators F_i satisfy

$$(2)\qquad [F_i, F_j] = if_{ijk}F_k\,,$$

where f_{ijk} is real and totally antisymmetric (2). The generators F_i are assumed to be expressible as integrals of currents j_i

$$(3)\qquad F_i = -i\int j_i^\mu(x)\,\mathrm{d}\sigma_\mu\,.$$

The currents $j_i(x)$ satisfy

$$(4)\qquad \frac{\partial j_i^\mu(x)}{\partial x^\mu} = \frac{\partial L}{\partial \varepsilon_i}\,,$$

where L is the lagrangian.

The matrices Λ_i must satisfy

$$(5)\qquad [\Lambda_i, \Lambda_j] = 2if_{ijk}\Lambda_k\,.$$

From invariance under the proper Lorentz group it follows that the matrix elements of Λ_i must belong to the group-algebra whose basic elements are 1 and γ_5. We write

$$(6)\qquad \Lambda_i = \Lambda_i^{(-)}a + \Lambda_i^{(+)}\bar{a}\,,$$

with $a = \frac{1}{2}(1+\gamma_5)$ and $\bar{a} = \frac{1}{2}(1-\gamma_5)$. Equation (5) is equivalent to

$$(6')\qquad [\Lambda_i^{(-)}, \Lambda_j^{(-)}] = 2if_{ijk}\Lambda_k^{(-)}\,,$$

$$(6'')\qquad [\Lambda_i^{(+)}, \Lambda_j^{(+)}] = 2if_{ijk}\Lambda_k^{(+)}\,.$$

Two possibilities arise:

1) $\Lambda_i^{(+)}$ and $\Lambda_i^{(-)}$ are equivalent irreducible three-dimensional representations of the Lie algebra of SU_3.

2) $\Lambda_i^{(+)}$ and $\Lambda_i^{(-)}$ are nonequivalent irreducible three-dimensional representations of the Lie algebra of SU_3.

We first deal with case 1). We choose the set $\Lambda_i^{(+)}$ to be a representation of the class $D^3(1, 0)$. The set $\Lambda_i^{(-)}$ must also be of the class $D^3(1, 0)$. The two sets are related by

$$(7)\qquad \Lambda_i^{(-)} = \Gamma\Lambda_i^{(+)}\Gamma^{-1}\,,$$

where Γ is nonsingular.

Charge conservation requires

$$(8)\qquad [\Gamma, Q] = 0\,,$$

where Q is the charge operator. Furthermore Γ must be nonsingular to preserve hermiticity and real to ensure time-reversal invariance.

The matrix Γ must then be either of the form

$$(9)\qquad \Gamma_A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\operatorname{sen}\omega \\ 0 & \operatorname{sen}\omega & \cos\omega \end{vmatrix}\,,$$

or of the form

$$(9')\qquad \Gamma_B = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \operatorname{sen}\omega \\ 0 & \operatorname{sen}\omega & -\cos\omega \end{vmatrix}\,.$$

For both cases A and B we give explicitly in Table I the form of the currents j_i.

We see immediately that if we want the currents to carry definite strangeness we must take $\operatorname{sen}\omega = 0$. But then $\cos\omega = \pm 1$ and the currents also have a definite character under parity.

Conventionally we assume the currents to have vector character. Then, if $\cos\omega = +1$: in case A, p, n, and Λ have the same parity; in case B, Λ has parity opposite to p and n. If $\cos\omega = -1$: in case A, p has parity opposite to n and Λ; in case B, n has parity opposite to p and Λ. The formalism yields all the four possible cases, but, of course, the relative parities between p, n, and Λ are only conventional.

The argument can be reversed: if the currents must have definite character under parity they must also carry definite strangeness.

We have thus completed the proof for case 1).

We now show that case 2) cannot be realized. The argument runs as follows.

We choose, as before, $\Lambda_i^{(+)}$ to be a representation of the class $D^3(1, 0)$. Then $\Lambda_i^{(-)}$ must be of the class $D^3(0, 1)$. We define the representation contragradient to $\Lambda_i^{(+)}$ as

$$\tilde{\Lambda}_i^{(+)} = -\Lambda_i^{(+)*}\,, \tag{10}$$

where the asterisk denotes complex conjugation.

Again we must have

$$\Lambda_i^{(-)} = \Delta\tilde{\Lambda}_i^{(+)}\Delta^{-1}\,, \tag{11}$$

where Δ is nonsingular.

Let us define a traceless charge operator

$$Q_0 = Q - \tfrac{1}{3}\operatorname{tr} Q\,. \tag{12}$$

The commutation relations of Q_0 are

$$[Q_0, \Lambda_i^{(+)}] = ig_{ik}\Lambda_k^{(+)}\,, \tag{13}$$

where $g_{ik} = f_{3ik} + (1/\sqrt{3})f_{8ik}$.

We must require that

$$[Q_0, \Lambda_i^{(-)}] = ig_{ik}\Lambda_k^{(-)}\,. \tag{14}$$

Let us define

$$Q_0' = \Delta^{-1}Q_0\Delta\,. \tag{15}$$

From (11) and (15) we obtain

$$[Q_0', \tilde{\Lambda}_i^{(+)}] = ig_{ik}\tilde{\Lambda}_k^{(+)}\,. \tag{16}$$

However

$$[\tilde{Q}_0, \tilde{\Lambda}_i^{(+)}] = ig_{ik}\tilde{\Lambda}_k^{(+)}\,. \tag{17}$$

We note that

$$\tilde{Q}_0 = -Q_0\,, \tag{18}$$

and sum (16) and (17). We get

$$[Q_0 + Q_0', \tilde{\Lambda}_i^{(+)}] = 0\,. \tag{19}$$

It follows from Schur's lemma that

$$Q_0 + Q_0' = cI\,, \tag{20}$$

where c is constant and I is the unit matrix. However Q_0 is traceless, and such is also Q_0', related to Q_0 by (15). Therefore $c=0$. Equation (20) can now be written as an anticommutator equation

$$\{\Delta, Q_0\} = 0\,. \tag{21}$$

From its definition (12) one sees that Q_0 has eigenvalues $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$; all different from zero and furthermore no pairs of them are equal and opposite. Then $\Delta=0$ is the only solution of (21).

This completes our proof for case 2).

The currents that one obtains by imposing either parity conservation or (equivalently) strangeness conservation also have the right isotopic-spin character.

If one relaxes the assumption of irreducibility for the representations, one has the two solutions: (a) $\Lambda_i^{(-)}=0$, $\Lambda_i^{(+)}$ belonging to $D^3(1, 0)$ or $D^3(0, 1)$; (b) $\Lambda_i^{(-)}$ belonging to $D^3(1, 0)$ or $D^3(0, 1)$, $\Lambda_i^{(+)}=0$. Both (a) and (b) lead to current with definite chiral character (right-handed for (a), left-handed for (b)) and with definite strangeness and isotopic spin.

The above derivation can be easily extended to the model based on G_2 and to the octet model. In both models parity conservation leads to conservation of strangeness and isotopic spin. In the model based on G_2 strangeness conservation leads to parity conservation and conservation of isotopic spin. In the octet model conservation of strangeness

3052

3053

TABLE I. – *Currents of the general Sakata model.* Cases A and B distinguish between the two possible choices for the sign of the determinant of the orthogonal transformation matrix Γ.

current	case A	case B
$\frac{1}{2}(j_1+ij_2)$	$-\frac{i}{2}[(\bar{p}\gamma n)+(\bar{p}\gamma a\Lambda)\sin\omega-(\bar{p}\gamma an)(1-\cos\omega)]$	
$\frac{1}{2}(j_1-ij_2)$	$-\frac{i}{2}[(\bar{n}\gamma p)+(\bar{\Lambda}\gamma ap)\sin\omega-(\bar{n}\gamma ap)(1-\cos\omega)]$	
j_3	$-\frac{i}{2}\{(\bar{p}\gamma p)-(\bar{n}\gamma n)-[(\bar{n}\gamma a\Lambda)+(\bar{\Lambda}\gamma an)]\sin\omega\cos\omega+[(\bar{n}\gamma an)-(\bar{\Lambda}\gamma a\Lambda)]\,\mathrm{sen}^2\,\omega\}$	
$\frac{1}{2}(j_4+ij_5)$	$-\frac{i}{2}[(\bar{p}\gamma\Lambda)-(\bar{p}\gamma an)\sin\omega-(\bar{p}\gamma a\Lambda)(1-\cos\omega)]$	$-\frac{i}{2}[-(\bar{p}\gamma\gamma_5\Lambda)+(\bar{p}\gamma an)\sin\omega+(\bar{p}\gamma a\Lambda)(1-\cos\omega)]$
$\frac{1}{2}(j_4-ij_5)$	$-\frac{i}{2}[(\bar{\Lambda}\gamma p)-(\bar{n}\gamma ap)\sin\omega-(\bar{\Lambda}\gamma ap)(1-\cos\omega)]$	$-\frac{i}{2}[-(\bar{\Lambda}\gamma\gamma_5 p)+(\bar{n}\gamma ap)\sin\omega+(\bar{\Lambda}\gamma ap)(1-\cos\omega)]$
$\frac{1}{2}(j_6+ij_7)$	$-\frac{i}{2}\{(\bar{n}\gamma\Lambda)-[(\bar{n}\gamma an)-(\bar{\Lambda}\gamma a\Lambda)]\sin\omega\cos\omega-$ $-[(\bar{n}\gamma a\Lambda)+(\bar{\Lambda}\gamma an)]\sin^2\omega\}$	$-\frac{i}{2}\{-(\bar{n}\gamma\gamma_5\Lambda)+[(\bar{n}\gamma an)-(\bar{\Lambda}\gamma a\Lambda)]\sin\omega\cos\omega+$ $+[(\bar{n}\gamma a\Lambda)+(\bar{\Lambda}\gamma an)]\sin^2\omega\}$
$\frac{1}{2}(j_6-ij_7)$	$-\frac{i}{2}\{(\bar{\Lambda}\gamma n)-[(\bar{n}\gamma an)-(\bar{\Lambda}\gamma a\Lambda)]\sin\omega\cos\omega-$ $-[(\bar{\Lambda}\gamma an)+(\bar{n}\gamma a\Lambda)]\sin^2\omega\}$	$-\frac{i}{2}\{-(\bar{\Lambda}\gamma\gamma_5 n)+[(\bar{n}\gamma an)-(\bar{\Lambda}\gamma a\Lambda)]\sin\omega\cos\omega+$ $+[(\bar{\Lambda}\gamma an)+(\bar{n}\gamma a\Lambda)]\sin^2\omega\}$
j_8	$-\frac{i}{2\sqrt{3}}\{(\bar{p}\gamma p)+(\bar{n}\gamma n)-2(\bar{\Lambda}\gamma\Lambda)+[(\bar{n}\gamma a\Lambda)+(\bar{\Lambda}\gamma an)]\cdot 3\sin\omega\cos\omega-[(\bar{n}\gamma an)-(\bar{\Lambda}\gamma a\Lambda)]\cdot 3\sin^2\omega\}$	

and of isotopic spin lead to conservation of parity. The physical fact for the different conclusion for the octet model is the degeneracy of Λ^0 and Σ^0, that are both taken as fundamental fields in that model.

The parity-conserving currents also have the appropriate isotopic-spin character. In the general case one would instead define a right-handed isotopic-spin and a left-handed isotopic spin, similarly as for the leptons that we have previously discussed [9]. The two kinds of isotopic spin coincide when parity conservation is imposed.

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(9) R. Gatto: *Nuovo Cimento*, **27**, 313 (1963).